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ON ANTIPLANE DEFORMATION FOR MATERIALS WHICH DO NOT OBEY HOOKE'S LAW

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Abstract—In the present paper the problems of antiplane deformation for a semiplane with two notches, for a semiplane with a hole, for a semiplane with periodically repeated notches and for a strip with two notches are investigated theoretically in the case of a particular nonlinear stress–strain dependence.

For linearization of equations we apply Chaplygin's transformation as Sokolovsky [5] has done it. The solutions are illustrated by numerical computations and plots.

1. INTRODUCTION

THE problems concerning prismatical bodies with longitudinal notches subjected to shear forces (i.e. the problems of so-called antiplane deformation) have drawn the attention of investigators in the case when the material of the body does not follow Hooke's law. This is explained by the fact that near notches the state of stress and strain taking place for antiplane deformation satisfactorily approximates that occuring under twisting of the prismatical body of corresponding section.

A number of elastic–plastic problems were solved theoretically. The problems for an angle with the flanges of equal width and for a plane with a circular hole were solved by Trefftz [1] in 1925. A method of solving the problems of prismatical bodies of polygonal cross section was proposed by Galin [2] in 1944. In 1957, Hult and McClintock [3] investigated the state of stress and strain for the semiplane with a V-notch. In 1962, Cherepanov [4] proposed a method of solution for the class of problems when the boundary consists of the straight and curved parts of lines, the straight parts being free of loading and the curvilinear arcs being completely enveloped by the plastic zone.

Further, the problems of antiplane deformation were considered for nonlinear elastic materials. In 1959, Sokolovsky [5] suggested a method for solving the problems of a special nonlinear 'stress-deformation' law, introduced by him in 1950, which assumed the solution of the corresponding problem in the case of Hooke's law being known. At the same time the problem of the motion of a plastic mass between two elliptical tubes was solved. The problem for the semiplane with an elliptical notch was solved by Sokolovsky [6] in 1962. In 1961, Neuber [7] studied a number of problems applying the same deformation law [5] but another method of solution.

Below, some problems for more complicated contours of cross section are solved by Sokolovsky's method [5]. The solutions obtained are illustrated by computations and plots. The calculations were carried out with the aid of a high-speed computer at the Computer Center of the Academy of Sciences of the U.S.S.R.

2. FORMULATION OF THE PROBLEM

We consider prismatic bodies, along whose generators uniform shear tractions are applied. The coordinate system is chosen in such a way that the x- and y-axes are in one of the cross sections of the prism. Only the component w of the displacement vector directed along the generator of the prism is different from zero. Nonvanishing components of the stress tensor τ_{xz} and τ_{yz} are denoted in the sequel by τ_x and τ_y, respectively. The quantities w, τ_x and τ_y depend upon the two coordinates x and y. In this case, only one of the equilibrium equations is not satisfied identically, namely

$$\frac{\partial \tau_x}{\partial x}+\frac{\partial \tau_y}{\partial y}=0. \tag{1}$$

The nonvanishing components of the deformation tensor γ_{xz} and γ_{yz} denoted below by γ_x and γ_y, respectively, are related to the displacement vector component w by the expressions

$$2\gamma_x=\frac{\partial w}{\partial x}, \qquad 2\gamma_y=\frac{\partial w}{\partial y}. \tag{2}$$

If all these assumptions are fulfilled, one says that the body is in a state of 'antiplane deformation'.

The relations between the stress tensor components and the strain tensor components are

$$\gamma_x=\frac{\gamma}{\tau}\tau_x, \qquad \gamma_y=\frac{\gamma}{\tau}\tau_y, \tag{3}$$

where τ and γ are determined by the formulae

$$\tau^2=\tau_x^2+\tau_y^2, \qquad \gamma^2=\gamma_x^2+\gamma_y^2. \tag{4}$$

The deformation law $\tau=\tau(\gamma)$ is assumed in the form

$$\tau=\frac{2k\gamma}{\sqrt{[1+(2m\gamma)^2]}}, \tag{5}$$

where k and m are parameters determining the mechanical behavior of the material; they should be taken from the experimental data. The curve $\tau-\gamma$ has an initial Young's modulus of $2k$ and a horizontal asymptote having a distance k/m from the γ axis. If $m=0$ Hooke's law holds, and if $k\to\infty$ and $k/m\to$ const. we have perfect plasticity. The dependencies τ vs. γ for different magnitudes of k with constant ratio k/m are given in Fig. 1.

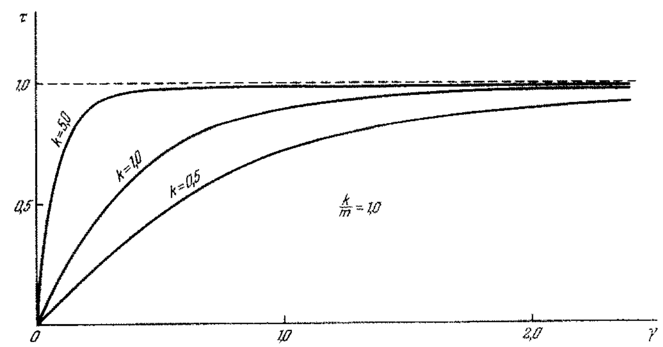

FIG. 1.

3. TRANSFORMATION OF EQUATIONS

The stress function $\psi(x, y)$ is introduced by the formulae

$$\tau_x = \frac{\partial\psi}{\partial y}, \qquad \tau_y = -\frac{\partial\psi}{\partial x}, \tag{6}$$

such that the equation of equilibrium (1) is satisfied, and after introducing the function

$$\phi = kw \tag{7}$$

the fundamental system of equations takes the form

$$\begin{aligned} \frac{\partial\phi}{\partial x} &= \frac{\tau_x}{\sqrt{[1-(m\tau/k)^2]}}, & \frac{\partial\phi}{\partial y} &= \frac{\tau_y}{\sqrt{[1-(m\tau/k)^2]}}, \\ \frac{\partial\psi}{\partial x} &= -\tau_y, & \frac{\partial\psi}{\partial y} &= \tau_x. \end{aligned} \tag{8}$$

In the case the boundary condition is

$$\psi(x, y) = 0, \tag{9}$$

as the stress vector touches the boundary contour, i.e. along the contour the following relation holds

$$\tau_x \, \mathrm{d}y - \tau_y \, \mathrm{d}x = 0. \tag{10}$$

Further, the equations are linearized by the method similar to Chaplygin's method in the problem of fluid jets. For this purpose the angle θ, formed by the vector τ and the x-axis, and an auxillary quantity t, which is the modulus of the stress vector in the case of Hooke's law, ($m = 0$), are introduced. The quantity t is connected with τ by the relation

$$\tau = \frac{t}{1+(nt)^2}, \tag{11}$$

where

$$n = \frac{m}{2k} \qquad \text{and} \qquad nt < 1. \tag{12}$$

We introduce the dimensionless variables

$$\begin{aligned} &x' = \frac{x}{l}, \qquad y' = \frac{y}{l}, \qquad \phi' = \frac{\phi}{kl}, \qquad \psi' = \frac{\psi}{kl}, \\ &t' = \frac{t}{k}, \qquad \tau' = \frac{\tau}{k}, \qquad n' = nk \end{aligned} \tag{13}$$

(l is a typical length) and primes will be omitted below. With the aid of the complex variables

$$\omega = \phi + i\psi, \qquad z = x + iy \tag{14}$$

the fundamental equations become

$$\mathrm{d}z = \mathrm{d}\zeta - n^2\left(\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}\bar{\zeta}}\right)^2 \mathrm{d}\bar{\zeta}, \tag{15}$$

$$\frac{d\omega}{d\zeta} = te^{-i\theta}. \tag{16}$$

Here the variable $\zeta = \xi + i\eta$ corresponds to the case $n = 0$, i.e. to Hooke's law.

The system of equations (15), (16), analogous to the equations of gas dynamics, permits the determination of the stress and displacement fields in the case of antiplane deformation with a nonlinear deformation law (5), if the solution of the corresponding problem for Hooke's law is known.

Indeed, if we know the function $\zeta = \zeta(\omega)$ from the solution of the linear problem, we can find the function $z = z(\omega, \bar{\omega})$ by means of integrating equation (15) and obtaining the solution to the given problem with the nonlinear deformation law (5). In this case, the contours, limiting the regions under consideration in the planes ζ and z, will be somewhat different. However, the solution contains arbitrary parameters by selection of which we can prescribe beforehand some typical dimensions of contours.

4. SOLUTION OF SOME PROBLEMS

4.1. *Semiplane with two notches*

Let us consider the problem of antiplane deformation of a prism, the cross section of which is a semiplane with two oval notches close to semicircles (Fig. 2). The shear stress $\tau_x = \tau_\infty$ acts at infinity. The complex potential, solving the corresponding linear problem is taken as

$$\omega(\zeta) = t_\infty\left[\zeta + \beta\left(\frac{1}{\zeta-\alpha} + \frac{1}{\zeta+\alpha}\right)\right] \tag{17}$$

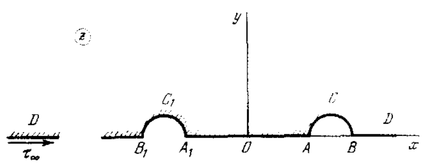

FIG. 2.

where t_∞, α and β are real parameters. Integration of equation (15) gives in this case

$$z = \zeta - n^2 t_\infty^2\left[\bar{\zeta} + 2\beta(\bar{v}_1 + \bar{v}_2) - \beta^2\left(\frac{\bar{\zeta}}{\alpha^2}\bar{v}_1\bar{v}_2 + \frac{1}{3}\bar{v}_1^3 + \frac{1}{3}\bar{v}_2^3 + \frac{1}{2\alpha^3}\ln\frac{\bar{v}_2}{\bar{v}_1}\right)\right], \tag{18}$$

or after separating real and imaginary parts

$$\begin{aligned} x = \xi - n^2 t_\infty^2 \Bigg\{ \xi + 2\beta(\lambda_1 + \lambda_2) - \beta^2 \Bigg[& \frac{1}{\alpha^2}(\xi\rho - \delta\eta) \\ & + \frac{1}{3}(\lambda_1^3 + \lambda_2^3 - 3\lambda_1\mu_1^2 - 3\lambda_2\mu_2^2) + \frac{1}{4\alpha^3}\ln\frac{\lambda_2^2+\mu_2^2}{\lambda_1^2+\mu_1^2}\Bigg]\Bigg\}, \\ y = \eta + n^2 t_\infty^2 \Bigg\{ \eta + 2\beta(\mu_1 + \mu_2) + \beta^2 \Bigg[& -\frac{1}{\alpha^2}(\eta\rho + \xi\delta) \\ & + \frac{1}{3}(\mu_1^3 + \mu_2^3 - 3\lambda_1^2\mu_1 - 3\mu_2\lambda_2^2) + \frac{1}{2\alpha^3}\left(\arctan\frac{\mu_1}{\lambda_1} - \arctan\frac{\mu_2}{\lambda_2}\right)\Bigg]\Bigg\}. \end{aligned} \tag{19}$$

Here the following notation is introduced:

$$\nu_{1,2} = \lambda_{1,2}+i\mu_{1,2}, \qquad \rho = \lambda_1\lambda_2-\mu_1\mu_2, \qquad \delta = \lambda_1\mu_2+\lambda_2\mu_1,$$
$$\lambda_{1,2} = \frac{\xi\mp\alpha}{(\xi\mp\alpha)^2+\eta^2}, \qquad \mu_{1,2} = \frac{-\eta}{(\xi\mp\alpha)^2+\eta^2}. \tag{20}$$

The values t and θ are determined by expressions

$$t^2 = t_\infty\{[1+\beta(\mu_1^2+\mu_2^2-\lambda_1^2-\lambda_2^2)]^2+4\beta^2(\lambda_1\mu_1+\lambda_2\mu_2)^2\},$$
$$\tan\theta = \frac{2\beta(\lambda_1\mu_1+\lambda_2\mu_2)}{1+\beta(\mu_1^2+\mu_2^2-\lambda_1^2-\lambda_2^2)}. \tag{21}$$

The maximum value of shear stress τ at any point is found from (11). The functions ϕ and ψ are determined by the equations

$$\phi = t_\infty[\xi+\beta(\lambda_1+\lambda_2)], \qquad \psi = t_\infty[\eta+\beta(\mu_1+\mu_2)], \tag{22}$$

and the longitudinal displacement will be

$$w = \frac{t_\infty}{k}[\xi+\beta(\lambda_1+\lambda_2)]. \tag{23}$$

In the ζ-plane, corresponding to the Hooke's law case, the boundary is determined by the equation

$$\psi(\xi,\eta) = 0. \tag{24}$$

The parametric equation of the boundary in the physical plane z is obtained by substituting the values of ξ, η, which satisfy equation (24), into equation (19). In the case under consideration in the ζ-plane, the rectilinear sections of the boundary DB_1, A_1A and BD are described by equation $\eta = 0$ and the curvilinear arcs $B_1C_1A_1$ and ACB by equation

$$[(\xi-\alpha)^2+\eta^2][(\xi+\alpha)^2+\eta^2] = 2\beta(\xi^2+\eta^2+\alpha^2). \tag{25}$$

These are the so-called Persey's curves which are the trace of the intersection of a torus with a plane which is parallel to its axis. We are interested in the case when these curves do not merge and do not touch each other. Therefore the parameters α and β must be subjected to the inequality

$$\alpha > \sqrt{\left[\frac{\beta}{2}(1+\sqrt{2})\right]}. \tag{26}$$

As there is a symmetry with respect to the η-axis in the ζ-plane and the y-axis in the z-plane, it is sufficient to discuss only points with positive abscissae.

The abscissae of points B and A will be

$$\xi_{1,2}^0 = \sqrt{[\alpha^2+\beta\pm\sqrt{(\beta^2+4\alpha^2\beta)}]}, \tag{27}$$

and the coordinates of point C of the curvilinear contour which is most distant from the ξ-axis have the following magnitudes

$$\xi^* = \sqrt{\left(\alpha^2-\frac{\beta^2}{4\alpha^2}\right)}, \qquad \eta^* = \sqrt{\left(\beta+\frac{\beta^2}{4\alpha^2}\right)}. \tag{28}$$

In the z-plane the curvilinear parts of the boundary will have a somewhat different form. However, the parameters α and β can be chosen in such a way as to fix, for example, the positions of points A and B.

Let us give a numerical example which illustrates the obtained solution. For computation the values $t_\infty = 1{\cdot}0$, $n = 0{\cdot}2$, $\alpha = 2{\cdot}1224$, $\beta = 1{\cdot}1155$ were chosen, and the abscissae of points A and B are then equal to 1·0 and 3·0, respectively. In this case the depth of the notch was 1·029 at the point with abscissa 2·082. The concentration factor reaches here the maximum value of 1·89, which is 10% less than in the case of one semicircular notch. The distribution of stress τ along the boundary contour and also along the straight lines $x = 0$, $x = 2{\cdot}082$ is shown in Fig. 3; the family of equal displacement lines ϕ = const. and the trajectories of stress ψ = const. is given in Fig. 4.

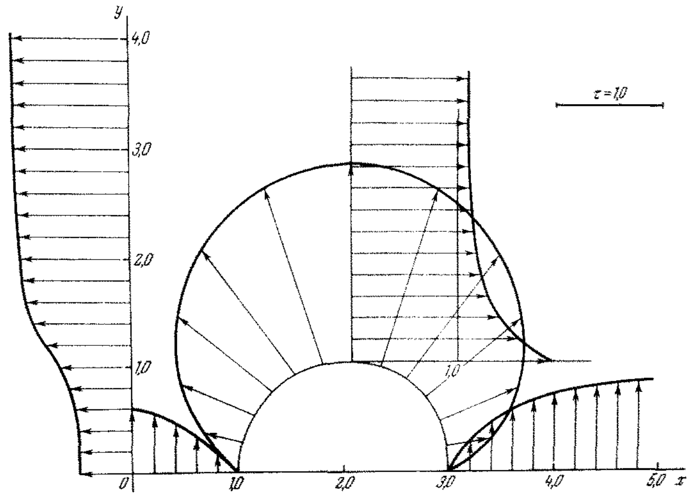

FIG. 3.

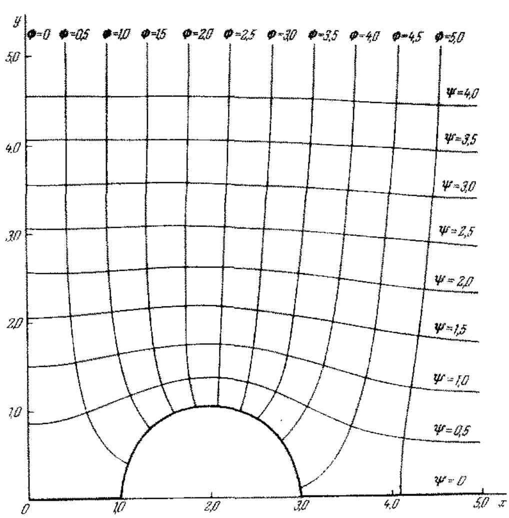

FIG. 4.

4.2. *Semiplane with an oval hole*

If we substitute in previous formulae $i\alpha$ instead of α, we shall obtain formulae giving the solution of the problem for the semiplane with an oval hole (Fig. 5).

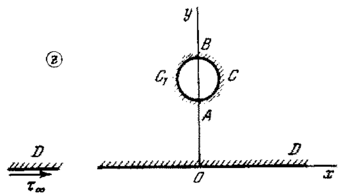

FIG. 5.

The relations (18), (21)–(23) do not change, but expressions (19) will become

$$x = \xi - n^2 t_\infty^2 \left\{ \xi + 2\beta(\lambda_1+\lambda_2) - \beta^2 \left[\frac{1}{\alpha^2}(\xi\rho - \delta\eta) + \frac{1}{3}(\lambda_1^3+\lambda_2^3-3\lambda_1\mu_1^2-3\lambda_2\mu_2^2) - \frac{1}{2\alpha^3}\left(\arctan\frac{\mu_1}{\lambda_1} - \arctan\frac{\mu_2}{\lambda_2}\right)\right]\right\}, \tag{29}$$

$$y = \eta + n^2 t_\infty^2 \left\{ \eta + 2\beta(\mu_1+\mu_2) + \beta^2 \left[-\frac{1}{\alpha^2}(\eta\rho + \xi\delta) + \frac{1}{3}(\mu_1^3+\mu_2^3-3\lambda_1^2\mu_1-3\lambda_2^2\mu_2) + \frac{1}{4\alpha^3}\ln\left(\frac{\lambda_2^2+\mu_2^2}{\lambda_1^2+\mu_1^2}\right)\right]\right\},$$

where ρ and δ are expressed in terms of λ_s and μ_s, as in formulae (19), but the functions λ_s and μ_s have another form, namely

$$\lambda_{1,2} = \frac{\xi}{\xi^2+(\eta\mp\alpha)^2}, \qquad \mu_{1,2} = \frac{\mp\alpha-\eta}{\xi^2+(\eta\mp\alpha)^2}. \tag{30}$$

4.3. *The strip with two notches*

Now we shall consider the problem of the strip with two oval notches close to semicircles (Fig. 6). The complex potential corresponding to the linear problem is taken as

$$\omega(\zeta) = t_\infty(\zeta + \beta \coth \alpha\zeta) \tag{31}$$

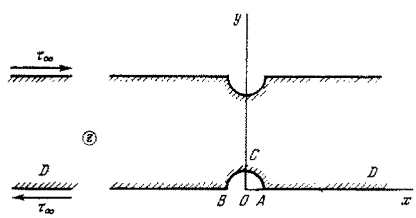

FIG. 6.

where t_∞, α and β are real parameters, which must be determined later. The integration of equation (19) gives

$$z = \zeta - n^2 t_\infty^2 \left[\bar{\zeta} + \beta(2+\alpha\beta)\bar{v} - \frac{\alpha\beta^2}{3}\bar{v}^3\right], \tag{32}$$

or, after separating the real and imaginary parts,

$$\begin{aligned} x &= \xi - n^2 t_\infty^2 \left[\xi + \beta(2+\alpha\beta)\lambda - \frac{\alpha\beta^2}{3}\lambda(\lambda^2 - 3\mu^2)\right], \\ y &= \eta + n t_\infty^2 \left[\eta + \beta(2+\alpha\beta)\mu - \frac{\alpha\beta^2}{3}\mu(3\lambda^2 - \mu^2)\right]. \end{aligned} \tag{33}$$

The following notation is introduced here

$$\begin{aligned} v &= \coth \alpha\zeta, \qquad v = \lambda + i\mu, \\ \lambda &= \frac{\sinh 2\alpha\xi}{2(\cosh^2 \alpha\xi - \cos^2 \alpha\eta)}, \qquad \mu = \frac{-\sin 2\alpha\eta}{2(\cosh^2 \alpha\xi - \cos^2 \alpha\eta)}. \end{aligned} \tag{34}$$

The quantities t and θ are determined by formulae

$$\begin{aligned} t^2 &= t_\infty^2\{[1+\alpha\beta(1+\mu^2-\lambda^2)]^2 + 4\alpha^2\beta^2\lambda^2\mu^2\}, \\ \tan\theta &= \frac{2\alpha\beta\lambda\mu}{1+\alpha\beta(1+\mu^2-\lambda^2)}. \end{aligned} \tag{35}$$

The modulus of the stress vector τ is found from equation (11), the longitudinal displacement w is

$$w = \frac{t_\infty}{k}(\xi + \beta\lambda) \tag{36}$$

and the functions ϕ and ψ are written in the form

$$\phi = t_\infty(\xi + \beta\lambda), \qquad \psi = t_\infty(\eta + \beta\mu). \tag{37}$$

In the ζ-plane the equation of the straight section of the contour will be $\eta = 0$, and the curvilinear section ACB is described by equation

$$\eta = \frac{\beta}{2} \frac{\sin 2\alpha\eta}{\cosh^2 \alpha\xi - \cos^2 \alpha\eta}. \tag{38}$$

The abscissae of points A and B have the form

$$\xi_{1,2}^0 = \pm\frac{1}{\alpha}\cosh^{-1}\sqrt{(1+\alpha\beta)}. \tag{39}$$

The ordinate of point C is found from equation

$$\eta^* = \beta \cot \alpha\eta^*. \tag{40}$$

The upper boundary contour of the strip is the mirror image of the lower one but for its determination the function $\psi(\xi, \eta)$ should be equated to a nonvanishing constant. To obtain the boundary contour in the z-plane, the corresponding values of ξ and η must be substituted into (19) and (20). By appropriately choosing the magnitudes of parameters α and β one can get, for example, the width of the strip and the depth of the notch.

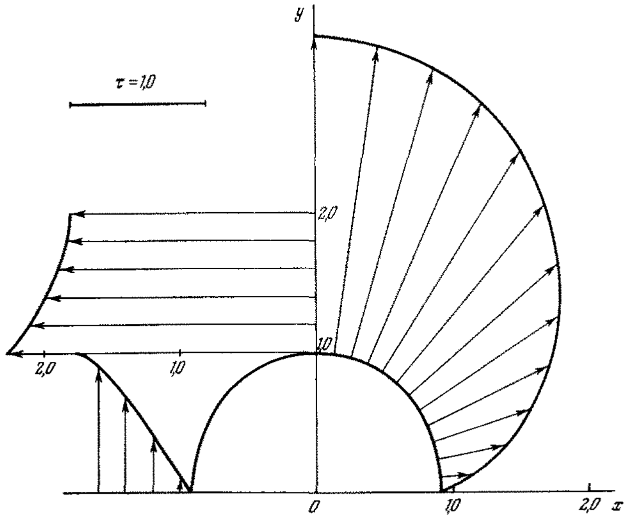

FIG. 7.

The solution obtained will be illustrated now with a numerical example. For calculations $t_\infty = 1{\cdot}0$, $n = 0{\cdot}2$, $\alpha = 0{\cdot}8168$, $\beta = 1{\cdot}4017$ are assumed. The width of the strip is then equal to four times the depth of the notch, and the notch diameter at the root is a factor of 1·82 times its depth. At the point C the concentration factor was a maximum and was equal to 2·265. In Fig. 7 the stress distribution τ along the contour and the straight line $x = 0$ was plotted. The family of lines of equal displacement $\phi = \text{const.}$ and the stress trajectories $\psi = \text{const.}$ are shown in Fig. 8.

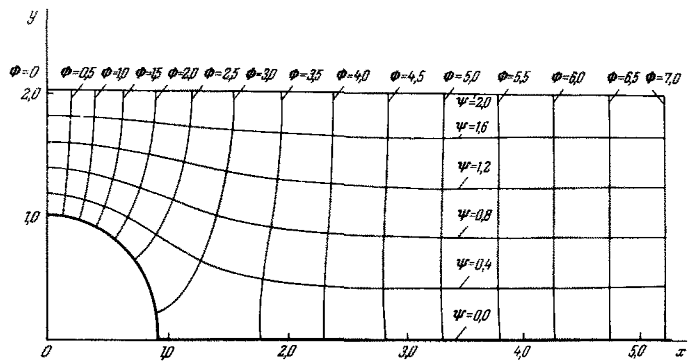

FIG. 8.

4.4. *Semiplane with periodically repeated notches* (*Fig. 9*)

The formulae, which represent the solution of the problem mentioned, can be obtained from the solution of the previous problem after the substitution of α by $i\alpha$. Then equation (31) will have the form

$$\omega(\zeta) = t_\infty(\zeta + \beta \cot \alpha\zeta). \tag{41}$$

The integral of equation (15) will be

$$z = \zeta - n^2 t_\infty^2 \left[\bar{\zeta} + \beta(2 - \alpha\beta)\bar{v} - \frac{\alpha\beta^2}{3}\bar{v}^3\right], \tag{42}$$

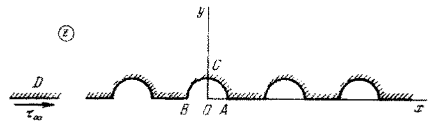

FIG. 9.

or, after separating the real and imaginary parts

$$x = \xi - n^2 t_\infty^2 \left[\xi + \beta(2-\alpha\beta)\lambda - \frac{\alpha\beta^2}{3}\lambda(\lambda^2 - 3\mu^2)\right],$$
$$y = \eta + n^2 t_\infty^2 \left[\eta + \beta(2-\alpha\beta)\mu - \frac{\alpha\beta^2}{3}\mu(3\lambda^2 - \mu^2)\right]. \tag{43}$$

However, v, λ and μ are here different, namely

$$v = \cot \alpha\zeta, \qquad v = \lambda + i\mu$$
$$\lambda = \frac{\sin 2\alpha\xi}{2(\sin^2 \alpha\xi + \sinh^2 \alpha\eta)}, \qquad \mu = \frac{-\sinh 2\alpha\eta}{2(\sin^2 \alpha\xi + \sinh^2 \alpha\eta)}. \tag{44}$$

The values t, θ and w are determined by the equations

$$t^2 = t_\infty^2\{[1 - \alpha\beta(1+\lambda^2-\mu^2)]^2 + 4\alpha^2\beta^2\lambda^2\mu^2\},$$
$$\tan\theta = \frac{2\alpha\beta\lambda\mu}{\alpha\beta(1+\lambda^2-\mu^2)-1}, \qquad w = \frac{t_\infty}{k}(\xi + \beta\lambda), \tag{45}$$

and the functions ϕ and ψ are expressed in terms of λ and μ with the aid of equations (37).

4.5. *Generalization*

If the complex potential of the linear problem for some notches has the form

$$\omega(\zeta) = t_\infty \left(\zeta + \sum_{s=1}^{\mathcal{N}} \frac{\beta_s}{\zeta - \alpha_s}\right), \tag{46}$$

where t_∞, α_s and β_s are parameters, the integration of the basic equation (15) gives

$$z = \zeta - n^2 t_\infty^2 \left\{\bar{\zeta} + 2\sum_{s=1}^{\mathcal{N}} \beta_s \bar{v}_s - \tfrac{1}{3}\sum_{s=1}^{\mathcal{N}} \beta_s^2 \bar{v}_s^2 - \sum_{\substack{s,r=1\\ s\neq r}}^{\mathcal{N}} \left[\frac{2\bar{\zeta} - (\bar{\alpha}_s + \bar{\alpha}_r)}{(\bar{\alpha}_s - \bar{\alpha}_r)^2}\beta_s\beta_r\bar{v}_s\bar{v}_r + \frac{2\beta_s\beta_r}{(\bar{\alpha}_r - \bar{\alpha}_s)^3}\ln\frac{\bar{v}_s}{\bar{v}_r}\right]\right\} \tag{47}$$

and equation (16) results in

$$t^2 = t_\infty^2\{[1 + \sum_{s=1}^{\mathcal{N}} (\mu_s^2 - \lambda_s^2)\beta_s]^2 + 4[\sum_{s=1}^{\mathcal{N}} \beta_s\lambda_s\mu_s]^2\},$$
$$\tan\theta = \frac{2\sum_{s=1}^{\mathcal{N}} \beta_s\lambda_s\mu_s}{1 + \sum_{s=1}^{\mathcal{N}} \beta_s(\mu_s^2 - \lambda_s^2)}. \tag{48}$$

The functions ϕ and ψ have the form

$$\phi = t_\infty(\xi + \sum_{s=1}^{\mathcal{N}} \beta_s \lambda_s), \qquad \psi = t_\infty(\eta + \sum_{s=1}^{\mathcal{N}} \beta_s \mu_s). \tag{49}$$

Here, for convenience, the following notation is introduced

$$\begin{gathered} \nu_s = \lambda_s + i\mu_s, \qquad \alpha_s = \alpha_s' + i\alpha_s'', \qquad s = 1, 2, \ldots, \mathcal{N} \\ \lambda_s = \frac{\xi - \alpha_s'}{(\xi - \alpha_s')^2 + (\eta - \alpha_s'')^2}, \qquad \mu_s = \frac{\alpha_s'' - \eta}{(\xi - \alpha_s')^2 + (\eta - \alpha_s'')^2}. \end{gathered} \tag{50}$$

In formulae (46)–(50) the number $\mathcal{N}$ is the number of holes or notches, t_∞ determines the external load; the parameters α_s and β_s characterize the geometry of the notches. Even though α_s and β_s influence both the diameters and the position of the notches, yet α_s determines the position of the notches to a considerable degree and β_s determines their diameters.

Formulae, representing the solution of problem 4.1 are obtained from the expressions (46)–(50), where $\mathcal{N} = 2$, $\alpha_1 = \alpha = -\alpha_2, \beta_1 = \beta_2 = \beta$ and $\alpha = \bar{\alpha}, \beta = \bar{\beta}$. Solution of the problem 4.3 can be obtained from the same formulae by passing to the limit as $\mathcal{N} \to \infty$.

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Zusammenfassung—In dem vorliegenden Aufsatz werden die Probleme der antiplanen Verformung im Falle einer besonderen, nicht-linearen Spannungs-und Beanspruchungsabhängigkeit theoretisch untersucht, und zwar für eine Halbebene mit zwei Einschnitten, für eine Halbebene mit einer Öffnung, für eine Halbebene mit periodisch sich wiederholenden Einschnitten und für einen Streifen mit zwei Einschnitten.

Die Linearisierung der Gleichungen erfolgt mit Hilfe der Chaplygin'schen Transformation, wie dieses bereits von Sokolovsky [5] durchgeführt wurde. Die Lösungen werden mittels zahlenmässiger Berechnungen und Kurven erläutert.

Абстракт В настоящей работе теоретически исследуются задачи об антиплоской деформации для полуплоскости с двумя выточками, для полуплоскости с отверстием, для полуплоскости с периодически повторяющимися выточками и для полосы с двумя выточками в случае специального вида нелинейной зависимости напряжение–деформация. Для линеаризации уравнений использовано преобразование Чаплыгина так, как это сделано Соколовским [5]. Решения иллюстрируются численными расчетами и графиками.